OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-3 : DIFFERENTIATION

UNIT TEST-1

1. If $\gamma(x) = \left(x^x\right)^x, x > 0$, then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to _____.

Hints and Solutions

1. Answer (16)

$\because y(x) = \left(x^x\right)^x$

$\therefore \quad y = x^{x^2}$

$\therefore \quad \frac{dy}{dx} = x^2 \cdot x^{x^2-1} + x^{x^2} \ln x \cdot 2x$

$\therefore \quad \frac{dx}{dy} = \frac{1}{x^{x^2+1}(1+2\ln x)}$ ...(i)

Now, $\frac{d^2x}{dx} = \frac{d}{dx}\left(\left(x^{x^2+1}(1+2\ln x)\right)^{-1}\right) \cdot \frac{dx}{dy}$

$= \frac{-x\left(x^{x^2+1}(1+2\ln x)\right)^{-2} \cdot x^{x^2}(1+2\ln x)\left(x^2+2x^2\ln x+3\right)}{x^{x^2}(1+2\ln x)}$

$= \frac{-x^{x^2}(1+2\ln x)\left(x^2+3+2x^2\ln x\right)}{\left(x^{x^2}(1+2\ln x)\right)^3}$

$\frac{d^2x}{dy^2(\text{at } x=1)} = -4$

$\therefore \quad \frac{d^2x}{dy^2(\text{at } x=1)} + 20 = 16$